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ANNIHILATION DIAGRAMS IN TWO-BODY NONLEPTONIC DECAYS OF CHARMED MESONS

P. Bedaque*, A. Das*
International Centre for Theoretical Physics
Trieste, Italy
and
V. S. Mathur
Department of Physics and Astronomy*
University of Rochester
Rochester, NY 14627

Abstract

In the pole-dominance model for the two-body nonleptonic decays of charmed mesons $D \to PV$ and $D \to VV$, it is shown that the contributions of the intermediate pseudoscalar and the axial-vector meson poles cancel each other in the annihilation diagrams in the chiral limit. In the same limit, the annihilation diagrams for the $D \to PP$ decays vanish independently.

*Permanent address

In a recent paper[1], we studied the two-body nonleptonic decays of charmed mesons using a simple pole-dominance model involving the vector, pseudoscalar and axial-vector mesons poles. A salient feature of this calculation in the decay to a final state of a pseudoscalar and a vector meson (PV) was the destructive interference between the pseudoscalar and the axial-vector meson pole contributions to the annihilation diagrams. In particular, in decays like $D^0 \to \phi \overline{K}^0$ and $D_S^+ \to \rho^+\pi^0$, which proceed only through annihilation diagrams, one finds large cancellations in the contributions of the pseudoscalar and axial-vector meson poles. This paper is devoted to a study of this interference effect.

For nonleptonic decay of charm, the effective weak Hamiltonian may be written as[2,3]

$$H_W = \frac{G_F}{\sqrt{2}} \left[a_1(\overline{u}d')_\mu(\overline{s}'c)_\mu + a_2(\overline{s}'d')_\mu(\overline{u}c)_\mu\right] \tag{1}$$

where $(\overline{q}^\alpha q_\beta)_\mu$ are color-singlet V-A currents

$$\left(\overline{q}^\alpha q_\beta\right)_\mu = i\overline{q}^\alpha\gamma_\mu\left(1+\gamma_5\right)q_\beta = (V_\mu)^\alpha_\beta + (A_\mu)^\alpha_\beta \qquad (\alpha,\beta = 1,2\ldots 4) \tag{2}$$

and $a_1,\ a_2$ are real coefficients which will be treated as phenomenological parameters. The primed quark fields are related to the unprimed ones by the usual Cabibbo-Kobayaski-Maskawa (CKM) mixing matrix.

In the pole-dominance model, we take the currents in H_W to be the hadronic currents given by the field-current identities $(\alpha,\beta = 1,2\ldots 4)$.

$$\begin{aligned} (V_\mu)^\alpha_\beta &= \sqrt{2}\ g_V(\phi_\mu)^\alpha_\beta \\ (A_\mu)^\alpha_\beta &= \sqrt{2}\ f_P\partial_\mu P^\alpha_\beta + \sqrt{2}\ g_A(a_\mu)^\alpha_\beta \end{aligned} \tag{3}$$

where $(\phi_\mu)^\alpha_\beta,\ P^\alpha_\beta$ and $(a_\mu)^\alpha_\beta$ are the field operators of the vector, the pseudoscalar and the axial-vector mesons, respectively, and $g_V,\ f_P$ and g_A are the corresponding decay constants. The nonleptonic weak interaction can then be represented by a two-meson vertex obtained on substituting (3) into (1).

The Feynman diagrams for the two-body decays $D \to PP,\ PV$ and VV can be readily drawn[1] in terms of the vector, pseudoscalar and axial-vector meson poles. The

pseudoscalar and axial-vector meson poles can contribute only to $D \to PV$ and $D \to VV$, but not to $D \to PP$. In Figs. 1 and 2, we display these contributions to the annihilation-type Feynman diagrams for $D \to PV$ and $D \to VV$, respectively. In these figures, the dark dot represents the weak vertex and the open circle the strong vertex. Also the dotted, solid and wavy lines represent the pseudoscalar, vector and axial-vector mesons, respectively.

The strong vertices appearing in Figs. 1 and 2 are of the type VPP, VPA, VVP and VVA. As in ref. 1, we use an extended spin-SU(4) symmetry to relate the VVP couplings to the VPP couplings. In fact we use a generalization of the Sakita-Wali interaction Hamiltonian[4], which relates the VPP, VVP and VVV couplings

$$\begin{aligned} H_{\rm str} = ig \ \mathrm{Tr} \Big(\phi_\mu P \overleftrightarrow{\partial_\mu} P - \frac{2}{M} \ \varepsilon_{\mu\nu\lambda\rho} P \partial_\mu \phi_\nu \partial_\lambda \phi_\rho \\ + \frac{2}{3} \ F_{\mu\nu} \phi_\mu \phi_\nu - \frac{2}{9M^2} \ F_{\mu\nu} F_{\nu\lambda} F_{\lambda\mu} \Big) \end{aligned} \tag{4}$$

Here the trace is over the SU(4) multiplets, g is a coupling constant and M represents a mass scale. We identify M with the mass of the decaying particle and take g to be the $\rho\pi\pi$ coupling determined from the decay $\rho \to 2\pi$. In our previous work, we described the VPA vertex phenomenologically, and neglected the VVA interaction. In the present work, we choose to describe the VPA and VVA vertices also in terms of the extended spin-SU(4) symmetry. Following[4] the arguments that led to (4), we obtain the interaction Hamiltonian in this case to be

$$\begin{aligned} H'_{\rm str} = ig' \ \mathrm{Tr} \Big(M \phi_\mu [P, a_\mu]_- + \frac{1}{4M} \ F_{\mu\nu} [P, F^a_{\mu\nu}]_- \\ - \frac{1}{2} \ \varepsilon_{\mu\nu\lambda\rho} \partial_\lambda \phi_\rho [\phi_\mu, a_\nu]_+ \Big) \end{aligned} \tag{5}$$

where $[X, Y]_\pm$ represent the anticommutator and the commutator, respectively, g' is a different coupling constant and

$$F_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu \quad , \quad F^a_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \tag{6}$$

are the field tensors.

Now, the coupling constants g and g' can be related through chiral symmetry. The simplest way to see this is to consider the matrix element of an axial-vector current between an ordinary vector and an ordinary pseudoscalar meson $< P|A_\mu|V >$. From Lorentz invariance, this can be written in the form

$$< P(k)|A_\mu(0)|V(p) > = \frac{i\varepsilon_\nu^V(p)}{(4p_0k_0V^2)^{1/2}} \left[K_1(q^2)\delta_{\mu\nu} + K_2(q^2)k_\nu(p+k)_\mu + K_3(q^2)k_\nu(p-k)_\mu\right] \tag{7}$$

where $K_{1,2,3}(q^2)$ are the form-factors and $q = p - k$. Chiral SU(3) symmetry, with massless pseudoscalar mesons, then implies

$$K_1(q^2) + m_V^2 K_2(q^2) - q^2 K_3(q^2) = 0 \tag{8}$$

We may now use the pole-dominance model to determine the form-factors in terms of the pseudoscalar and axial-vector meson poles. The corresponding Feynman diagrams are shown in Fig. 3. The dash-and-dot line represents the axial-vector current and the vertices denoted by crosses can be read off from the field-current identity in Eq. (3). We find

$$\begin{aligned} K_1(q^2) &= -2M\frac{g_A g'}{q^2+m_A^2}\ \left[1+\frac{1}{4M^2}\ \left(q^2-m_V^2\right)\right] \\ K_2(q^2) &= -\frac{1}{2M}\ \frac{g_A g'}{q^2+m_A^2} \\ K_3(q^2) &= \frac{4f_P g}{q^2} + \frac{2M}{m_A^2}\ \frac{g_A g'}{q^2+m_A^2}\ \left(1-\frac{m_A^2}{4M^2}\right) \end{aligned} \tag{9}$$

Substituting (9) in (8), we readily obtain

$$2f_P g + \frac{g_A}{m_A^2}\ Mg' = 0 \tag{10}$$

Returning to the decays $D \to PV$ and $D \to VV$, we define the decay amplitudes as follows

$$M\left(D(q) \to V_1\left(q_1,\lambda_1\right)P_2\left(q_2\right)\right) = \frac{-i(2\pi)^4\delta^{(4)}(q-q_1-q_2)}{\sqrt{2q_0V2q_{10}V2q_{20}V}}q_2\cdot\varepsilon^{(\lambda_1)}(q_1)B \tag{11}$$

$$M\big(D(q)\to V_1(q_1,\lambda_1)V_2(q_2,\lambda_2)\big) = \frac{-i(2\pi)^4\delta^{(4)}(q-q_1-q_2)}{\sqrt{2q_0V2q_{10}V2q_{20}V}}$$
$$\big[iC\delta_{\alpha\beta}+iD\varepsilon_{\mu\alpha\nu\beta}q_{1\mu}q_{2\nu}+iEq_{1\beta}q_{2\alpha}\big]\varepsilon_\alpha^{(\lambda_1)}(q_1)\varepsilon_\beta^{(\lambda_2)}(q_2) \tag{12}$$

The contributions of the pseudoscalar and axial-vector meson poles to the annihilation diagrams in Fig. 1 for the decay $D\to PV$ are easily calculated to be

$$\begin{aligned} B_{\rm ann}(P) &= -4\ \alpha\ a\ f_D f_P\ \frac{m_D^2}{m_P^2-m_D^2}\ g \\ B_{\rm ann}(A) &= 2\ \alpha\ a\ f_D \frac{g_A}{m_A^2} M g' \end{aligned} \tag{13}$$

where a stands for the coefficient a_1 or a_2 in the weak Hamiltonian (1), depending on which term in (1) contributes, and

$$\alpha = \frac{G_F}{\sqrt{2}}\ V_{cs}^* V_{ud} \tag{14}$$

In (14), the V's represent the matrix elements of the CKM matrix. Here we are considering the Cabibbo allowed decays, but the considerations can be readily extended to the Cabibbo suppressed decays also. Note that in Fig. 1, P is an ordinary pseudoscalar meson, so that in the chiral limit with $m_P\to 0$, we get from (13), on using the result (10)

$$B_{\rm ann}(P)+B_{\rm ann}(A)=0 \tag{15}$$

Thus, in the chiral limit, the pseudoscalar and axial-vector meson pole terms cancel each other in the annihilation diagrams of the decay $D\to PV$.

For the decay $D\to VV$, we similarly find the contributions of the pseudoscalar and axial-vector meson poles to the annihilation diagrams in Fig. 2 to be

$$\begin{aligned} C_{\rm ann}(P) &= E_{\rm ann}(P)=0 \\ D_{\rm ann}(P) &= 4\ \alpha\ a\ f_D f_P\ \frac{m_D^2}{m_P^2-m_D^2}\ \frac{g}{M} \\ C_{\rm ann}(A) &= E_{\rm ann}(A)=0 \\ D_{\rm ann}(A) &= -2\ \alpha\ a\ f_D \frac{g_A}{m_A^2} g' \end{aligned} \tag{16}$$

Once again in the chiral limit, we find on using (10)

$$D_{\text{ann}}(P) + D_{\text{ann}}(A) = 0 \tag{17}$$

Thus, in the chiral limit, the pseudoscalar and axial-vector meson pole contributions to the annihilation diagrams in $D \to VV$ also cancel each other.

It should also be noted that in the decay $D \to P_1P_2$, where only the vector meson pole contributions, the annihilation diagram also vanishes in the chiral limit. This is easily seem from the result[5]

$$A_{\text{ann}} \propto \frac{m_2^2 - m_1^2}{m_V^2} \tag{18}$$

for the vector meson contribution to the annihilation amplitude in $D \to P_1P_2$, where $m_1,\ m_2$ are the masses of the pseudoscalar mesons in the final state.

It should be remarked that the vanishing of the annihilation amplitudes in the chiral limit is not surprising. Indeed, it is the analogue of the helicity suppression of the annihilation amplitude in the quark model. In fact, for massless quarks (in the chiral limit), it is well-known that the quark model annihilation amplitude vanishes. Also our result is consistent with the result of Bauer et al.[3] for the suppression of annihilation amplitude in the factorization model.

With g' determined in the chiral limit from Eq. (10), we could calculate all two-body decays of the D mesons in the pole-dominace model[1] just in terms of the parameters a_1 and a_2 appearing in the weak Hamiltonian (1). However, in view of the sensitivity of the destructive interference effect discussed above to small variations in g', this is not the best approach. It should be noted that uncertainties in the determination of g' from Eq. (10) arise not only due to the use of chiral symmetry but also the choice of the mass scale M. In view of this, the phenomenological approach to the calculation of the decay rates followed in ref. 1 is preferable.

Finally, we can calculate the width for the decay $A_1 \to \rho\pi$ which also depends on g'.

Using (10), we find $g' = -6.5$, which gives

$$\Gamma(A_1 \to \rho\pi) = 566 \text{ MeV} \tag{19}$$

Experimentally, this width is not well-determined but our result may be compared with the value quoted in the particle properties data booklet[6] of ~400 MeV.

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References and Footnotes

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Figure Captions

Fig. 1 Feynman diagrams for the pseudoscalar and axial-vector meson pole contributions to the annihilation amplitude in the decay $D \to PV$.

Fig. 2 Feynman diagrams for the pseudoscalar and axial-vector meson meson pole contributions to the annihilation amplitude in the decay $D \to VV$.

Fig. 3 Feynman diagrams for the pseudoscalar and axial-vector meson pole contributions to the form-factors in Eq. (7).